Somes exercices for microeconomics

Part I

Exercise 1
Graph a typical indifference curve for the following utility functions, and determine whether they have convex indifference curves (i.e. whether MRS declines as $x_1$ increases

1. $U(x, y) = 3x_1 + x_2$
2. $U(x, y) = x_1^{1/2}x_2^{1/2}$
3. $U(x, y) = x_1^{1/2} + x_2$
4. $U(x, y) = \frac{x_1x_2}{x_1 + x_2}$

Brief Answer
1. No
2. Yes
3. Yes
4. Yes

Exercise 2
Consider the quasi-concave utility function $U(x_1, x_2) = x_1 + \ln x_2$. This is a function that is used relatively frequently in economic modeling as it has some useful properties

1. Find the MRS of the function. Interpret the result.
2. Find the equation for an indifference curve for this function
3. Compare the marginal utility of $x_1$ and $x_2$. How do you interpret theses functions?

How might consumers choose between $x_1$ and $x_2$ as they try to increase their utility by consuming more when their income increases?

Brief Answer
1. $MRS = U_{x_1}/U_{x_2} = x_2$
2. IC function : $x_2 = \exp(\bar{U} - x_1)$
3. Marginal utility of $x_1$ is constant, marginal utility of $x_2$ diminishes. As income rises, consumers will eventually choose only added $x_2$

Exercise 3
Consider a CES (constant elasticity of substitution) utility function $U(x_1, x_2) = (x_1^\delta + x_2^\delta)^{1/\delta}$
1. The indirect utility function is the utility function evaluated at the individual optimal choice. Show that the indirect utility function for the utility function just given is \( V = R \left( p_{x_1}^r + p_{x_2}^r \right)^{-1/r} \) where \( r = \delta / (\delta - 1) \), \( R \) is income, and \( p_{x_1}, p_{x_2} \) are prices of good 1 and 2.

2. Show that the function \( V \) is homogeneous of degree 0 in prices and income.

3. Show that the function \( V \) is strictly increasing in income.

4. Show that the function \( V \) is strictly increasing in any price.

5. The expenditure function for this CES utility function is given by \( R = V \left( p_{x_1}^r + p_{x_2}^r \right)^{1/r} \).
   Show that this function is homogeneous of degree 1 in the goods’ prices, increasing and concave in each price.

**Exercise 4**
Suppose that a firm’s fixed proportion production function is given by \( y = \min(5k, 10l) \) where \( k \) represents capital and \( l \) labor. Capital price is \( r \) and labor price is \( w \).

1. Calculate the firm’s long-run total, average and marginal cost functions
2. Suppose that \( k \) is fixed at 10 in the short run. Calculate the firm’s short-run total, average and marginal cost functions

**Brief Answer**

1. \( C_l = y \left( r/5 + w/10 \right) \)  
   \( AC_l = MC_l = (r/5 + w/10) \)
2. For \( y \leq 50 \), \( C_s = 10r + wy/10 \)  
   \( AC_s = 10r/y + w/10 \)  
   \( MC_s = w/10 \)

**Part II**

**Exercise 5**
Suppose that there are 100 identical firms in perfectly competitive industry. Each firm has a short-run total cost function of the form

\[ C(y) = \frac{1}{300} y^3 + 0.2y^2 + 4y + 10 \]

1. Calculate the firm’s short-run supply curve with \( y \) as a function of market price \( p \).
   Calculate the short-run industry supply curve \( Y_s \).
2. Suppose market demand demand is given by \( Y_d = -200p + 8000 \). What will be the short-run equilibrium price-quantity combination?

**Exercise 6**
A perfectly competitive industry has a large number of potential entrants. Each firm has an identical cost structure such that long-run average cost is minimized at an output of 20 units \( (y_i = 20) \). The minimum average cost is $10 per unit. Total market demand is given by

\[ Y_d = 1500 - 50P \]
1. What is the industry’s long-run supply schedule?

2. What is the long-run equilibrium price? The total industry output at the long-run equilibrium? The number of firm? The profit of each firm?

3. The short-run total cost function associated with each firm’s long-run equilibrium output is given by

\[ C = 0.5y^2 - 10y + 200 \]

Calculate the short-run average and marginal cost function. At what output level does short-run average cost reach a minimum?

4. Calculate the short-run supply function for each firm and the industry short-run supply function.

5. Suppose now that the market demand function shifts upward to \( Y_d = 2000 - 50P \). Using this new demand curve, calculate the new short-run and long-run equilibrium for the industry.

**Exercise 7**

A single firm monopolizes the entire market for widgets and can produce at constant average and marginal cost \( AC = MC = 10 \). The firm faces a market demand curve given by

\[ Y_d = 60 - P \]

1. Calculate the profit-maximizing price-quantity combination for the firm and its profit.

2. Now assume that the market demand curve shifts outward, becoming steeper and is given by

\[ Y_d = 45 - 0.5P \]

What is the firm’s profit-maximizing price-quantity combination and its profit now?

3. Assume that the market demand curve shifts outward, becoming flatter and is given by

\[ Y_d = 100 - 2P \]

What is the firm’s profit-maximizing price-quantity combination and its profit now?

4. Graph the three different situations. Comments.

**Brief Answer**

1. \( Y = 25, P = 35, \pi = 625 \)

2. \( Y = 20, P = 50, \pi = 800 \)

3. \( Y = 40, P = 30, \pi = 800 \)
Exercise 8
Let $c_i$ be the constant marginal and average cost for firm $i$. Suppose inverse demand is given by $P = 1 - Y_d$

1. Calculate the best-response function for each firm assuming there are two firms in a Cournot market.

2. Calculate the Nash equilibrium quantities $y_1, y_2$. Also calculate market output, market price, firm profits, industry profit, consumer surplus and social surplus.

3. Represent the Nash-Cournot equilibrium on a best-response function diagram $(y_2, y_1)$

Brief Answer

1. Maximize firm’s profit

2. Equilibrium quantities are: $y_i = (1 - 2c_i + c_j)/3$. Market output is $Y = (2 - c_1 - c_2)/3. P = (1 + c_1 + c_2)/3. \pi_i = (1 - 2c_1 + c_2)^2/9$. Consumer surplus = $(2 - c_1 - c_2)^2/18$. Social surplus = total profit + consumer surplus as fixed cost is null.

Exercise 9
Suppose that firms’ marginal and average costs are constant and equal to $c$ and that inverse market demand is given by $P = a - bY$ where $a, b > 0$.

1. Calculate the profit-maximizing price-quantity combination for a monopolist, and its profit.

2. Calculate the Nash equilibrium quantities for Cournot duopolists, which choose quantities for their identical products simultaneously. Also compute market output, market price as well as firm and industry profits.

3. Calculate the Nash equilibrium prices for Bertrand duopolists, which choose prices for their identical products simultaneously. Also compute firm and market output, as well as firm and industry profits.

4. Suppose now that there are $N$ identical firms in a Cournot model. Compute the market output, and market price at the Nash-Cournot equilibrium as functions of $N$.

5. Show that the monopoly outcome from question (1) can be reproduced in question (4) by setting $N = 1$, that the Cournot duopoly outcome from question (2) can be reproduced in question (4) by setting $N = 2$, and that letting $N$ approach infinity yields the same market price, output and industry profit as in question (3).

Brief Answer

4 Inverse market demand is written as $P = a - b \sum_{n=1}^{N} y_n$. Each firm maximizes its profit function. FOC for a Nash-Cournot equilibrium are:

$$a - b \sum_{n=1}^{N} y_n - by_i - c = 0$$
Adding these FOC for N firms yields:

\[ Na - Nb \sum_{n=1}^{N} y_n - b \sum_{i=1}^{N} y_i - Nc = 0 \]

Then, market output is given by

\[ Y = \sum_{i=1}^{N} y_i = \frac{N}{N + 1} \frac{a - c}{b} \]

**Part III**

**Exercise 10**

Suppose there are three goods \( (x_1, x_2, x_3) \) in an economy and that the excess demand functions for \( x_2 \) and \( x_3 \) are given by

\[
Z_2 = -\frac{3p_2}{p_1} + \frac{2p_3}{p_1} - 1
\]

\[
Z_3 = -\frac{4p_2}{p_1} - \frac{2p_3}{p_1} - 2
\]

1. Show that these functions are homogenous of degree 0 in all prices.

2. Use Walras’ law to show that, if \( Z_2 = Z_3 = 0 \), then \( Z_1 \) must also be 0. Can you also use Walras’ law to calculate \( Z_1 \)?

3. Solve this system of equations for the equilibrium relative prices \( p_2/p_1 \) and \( p_3/p_1 \).

**Brief Answer**

1. Doubling prices leaves excess demand unchanged.

2. By Walras’ law \( p_1Z_1 + p_2Z_2 + p_3Z_3 = 0 \) for all prices. Then, \( Z_2 = Z_3 = 0 \), implies that \( p_1Z_1 = 0 \) and \( Z_1 = 0 \). Then \( Z_1 \) can be calculated as

\[
Z_1 = \frac{(3p_2^2 - 6p_2p_3 + 2p_3^2 + p_1p_2 + 2p_1p_3)}{p_1^2}
\]

This is also homogenous of degree 0 in all prices.

3. \( p_2/p_1 = 3, p_3/p_1 = 5 \)

**Exercise 11**

Consider a simple two-person, two-good exchange economy. Suppose that total quantities of the goods are fixed at \( \omega_1 = \omega_2 = 1000 \). Person A’s utility takes the Cobb-Douglas form:

\[ U(x_{1A}, x_{2A}) = x_{1A}^{2/3} x_{2A}^{1/3} \]

and person B’s utility is given by

\[ U(x_{1B}, x_{2B}) = x_{1B}^{1/3} x_{2B}^{2/3} \]

1. Calculate the contract curve function

2. Consider a Pareto optimal allocation with \( x_{1A} = 500, x_{2A} = 200, x_{1B} = 500, x_{2B} = 800 \). Calculate the equilibrium price ratio at this point on the contract curve.
Part IV

Exercise 12
Consider an agency relationship in which the principal contracts the agent, whose effort determines the result. Assume that the uncertainty present is represented by three states of nature. The agent can choose between two effort levels. The results are shown in following table

<table>
<thead>
<tr>
<th>states of Nature</th>
<th>$\varepsilon_1$</th>
<th>$\varepsilon_2$</th>
<th>$\varepsilon_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$e = 6$</td>
<td>60000</td>
<td>60000</td>
<td>30000</td>
</tr>
<tr>
<td>$e = 4$</td>
<td>30000</td>
<td>60000</td>
<td>30000</td>
</tr>
</tbody>
</table>

The principal and the agent both believe that the probability of each state is $1/3$. The objective functions of the principal and the agents are, respectively:

$$B(x, w) = x - wU(w, e) = w^{1/2} - e^2$$

where $x$ is the monetary result of the relationship and $w = w(x)$ is the wage that the agent receives. Assume that the agent will only accept the contract if he obtains an expected utility level of at least 114 (his reservation utility function).

1. What can be deduced from the participants’ objective functions?

2. What would be the effort and the wage in a situation of symmetric information? What would happen if the principal were not risk-neutral?

3. What happens in a situation a symmetric information? What wage scheme allows an effort level of $e = 4$ to be obtained? What wage schema allows the effort level of $e = 6$ to be obtained? Which effort level does the principal prefer? Discuss the result.

Brief Answer

1. The principal is risk-neutral and the agent is risk-averse

2. The optimal contracts are derived from (i) the principal accepts all the risk, and (ii) the PC binds.

   If $e = 6$, then $w$ is such that $w^{1/2} - 6^2 = 114$, then $w = 22500$. In this case, $U = 50000 - 22500 = 27500$.

   If $e = 4$, then $w = 16900$ and $U = 23100$.

   The symmetric solution is then $e^* = 6, w^* = 22500$

   If the principal was not risk-neutral, then both participant would share the risk inherent in the relationship.

3. The optimal contract if $e = 4$ is the same as before since given a constant wage the agent will always choose the lowest effort level. In order to achieve $e = 6$, the principal must offer a contract that is contingent on the result. she will pay $w(60)$ if the result is 60000 and $w(30)$ is the result is 30000.
The contract must simultaneously satisfy the PC and the IC

\[(PC) \frac{2}{3}w(60)^{1/2} + \frac{1}{3}w(30)^{1/2} - 36 \geq 114\]

\[(IC) \frac{2}{3}w(60)^{1/2} + \frac{1}{3}w(30)^{1/2} - 36 \geq \frac{1}{3}w(60)^{1/2} + \frac{2}{3}w(30)^{1/2} - 16\]

It is shown that both constraints bind (see chapter 8, pp 8-10)
The solution is \(w(60) = 28900\) and \(w(30) = 12100\). The principal expected profit \(B\) is 26700.

Under asymmetric information the principal also chooses \(e = 6\), since \(26700 > 23100\), but an efficiency loss measured by the reduction in the expected profits of the principal (the agent always obtains his reservation utility)

**Exercise 13**
Assume that a worker can exert two effort levels, good or bad, which induce a production error with probability 0.25 and 0.75 respectively. His utility function is

\[U(w, e) = 100 - 10/w - v\]

where \(w\) is the wage received and \(v\) takes the value 2 if effort is good and 0 if effort is bad. Production errors are observable but effort cannot. The product obtained is worth 20 if there are no errors and 0 otherwise. Assume that the worker has reservation utility equal to \(U = 0\).
The principal is risk-neutral. His objective function is

\[B(x, w) = x - w\]

1. Calculate the optimal contract and the effort that the principal desires under conditions of symmetric information on the agent’s behavior.

2. Calculate the optimal contract and the effort that the principal desires under conditions of symmetric information on the agent’s behavior.

**Brief Answer**
Two effort levels \(e_h > e_l\)
Two effort costs \(v(e_h) = 2, v(e_l) = 0\)
For \(e_h\), \(p_1(e_h) = \text{probability (error}/e_h) = 0.25\), then \(p_2(e_h) = \text{probability (non error}/e_h) = 0.75\),
For \(e_l\), \(p_1(e_l) = \text{probability (error}/e_l) = 0.75\), then \(p_2(e_l) = \text{probability (non error}/e_l) = 0.25\),
Two results: \(x_2 = 20, x_1 = 0\)

1. It is shown that under conditions of symmetric information, when the principal is risk-neutral and the agent is risk-averse, the optimal contract indicates that (i) the principal accepts all the risk (she gives then a constant wage to the agent) and (ii) the agent’s participation constraint binds. Therefore, optimal wage is determined from saturated participation constraint.
If the principal demands low effort, she will pay a constant wage of $w = 1/10$ to the agent. This is true both under symmetric and asymmetric information. The principal’s profit is $B = 0.2520 - 1/10 = 49/10$. If the principal demands high effort, and if this effort is contractual, she will offer a constant wage of $w = 10/98$. The principal’s profit in this case is $B = 0.7520 - 10/98 = 14.988$

2. When there is asymmetric information on effort, the principal should offer a contract $(w_1, w_2)$ that depends on success or failure. The contract should satisfy both the participation and incentive constraints when the effort exerted is high. It is shown that the two constraints must bind (see chapter 8, page 8-10)

The optimal contract $(w_1, w_2)$ with high effort are determined from saturated participation and incentive constraints.

\[
(PC) \quad 100 - \frac{2.5}{w_1} - \frac{7.5}{w_2} = 2 \Rightarrow \frac{1}{w_1} + \frac{3}{w_2} = \frac{392}{10}
\]

\[
(IC) \quad \frac{5}{w_1} - \frac{5}{w_2} = 2 \Rightarrow \frac{1}{w_1} - \frac{1}{w_2} = \frac{4}{10}
\]

The solution is $w_2 = 10/97, w_1 = 10/101$. The expected profit of the principal is $B = \frac{3}{4}(20 - 10/97) - \frac{1}{4}(10/101)$, which is greater than that obtained with low effort.