Exercise 1 (2 points)
True or False. In this exercise, justify whether each statement is True or False. You should explain your answer.

1. After a professor promises that no one would fail, students never study. This is an example of adverse selection problem.

2. Because all points on a contract curve are efficient, they are all equally desirable from a social point of view.

3. At the general (Walrasian) equilibrium, only relative prices can be determined.

4. A steel plant that pollutes a river has negative externalities on residents along the river. If the steel and residents can negotiate, an efficient result can be achieved only if the pollution right is assigned to the steel plant.
Exercise 2 (2 points)
State the First Theorem of Welfare Economics. What are its implications and limits?

Exercise 3 (3 points)
Research and Development (R&D) subsidies can be studied in a moral hazard framework. There are many reasons for the government to intervene in the private R&D activities of firms. It is often argued that R&D is a risky activity and that it generates positive externality. This means that in general, the private gains from carrying-out R&D are less than the social gains, in other words, too little research is undertaken by firms. This is why it is often necessary for the government to subsidize firms that are involved in R&D processes. The objective of the subsidies policy is to give the firms incentives to undertake R&D to a greater degree than they would without public intervention.

Now, it is also well known that R&D is a very difficult process to control, that is, it is very difficult for the government to know exactly how much money a given firm is dedicating to this activity. A subsidy is a contract between the government (the principal) and a firm (the agent), for the firm to exert an effort (dedicate money to R&D) greater than
that which it would spontaneously (without a subsidy). The government obtains gain from the result (the final technology level), since a better technology is useful for other firms, and beneficial to consumers. The technological advances achieved are the result of an investment of effort and random components.

1. Where does the moral hazard problem appear in this context? In which case would there be no moral hazard problem? (1 point)

2. Consider a subsidy that consists of giving money to a firm independently of the final result of the research. Comment on this subsidy policy. What would be the effect on the firm's decisions? (1 point)

3. The concession of patent is often taken as a sign of research success. Imagine that the government gives a subsidy that consists of giving more money to the firm if it gets idea patented. Comment on this subsidy policy. Is it better or worse than the previous one? (1 point)

Exercise 4 (5 points)
Consider a perfectly competitive industry. The market demand is given by

\[ Y_d = 100 - 5p \]

where \( p \) is the price of the good.

I) Suppose that there are 40 identical firms. Each firm has a cost function of the form:

\[ C(y) = y^2 + 1 \]

where \( y \) is the firm's output.
1. Calculate the short-run supply function for each firm and the industry short-run supply function. (1 point)

2. Calculate the short-run competitive equilibrium (price and total output). What are the output and the profit of each firm? (1 point)

II) This competitive market has a large number of potential entrants. Each potential entrant has the same cost function than firms already present on the market.

3. Calculate the long-run competitive equilibrium (price and total output). What is the number of firms? (1 point)

4. Calculate the surplus of consumers and the social surplus. (1 point)
III) Now suppose that the market demand function shifts upward to

\[ Y_d = 200 - 5p \]

5. Using this new demand curve, calculate the new short-run and long-run equilibrium for the industry. Comments (1 point)

Exercise 5 (4 points)
Consider a simple model in which there are two individuals, A and B. There are also two goods. Good \( x \) is an ordinary private good, each person begins with an allocation of this good given by \( x^A \) and \( x^B \), respectively. Suppose that each individual may choose to consume some of his initial endowment \( x^* \) directly or to devote some portion of it to the production of a pure public good, \( G \). The amounts contributed are given by \( x^A \) and \( x^B \), and the public good is produced according to the production function:

\[ G = f(x^A + x^B) \]
Utilities for these two individuals are given by

\[ U^A(G, (x^{A*} - x^A)) \]
\[ U^B(G, (x^{B*} - x^B)) \]

1. In a centralized economy, a social planner may choose \( x^A \) and \( x^B \) maximizing a social welfare function. Write the first order conditions for a utilitarian optimum allocation, and deduce from them the optimality (Samuelson) condition for producing the public good. Interpret this condition. (2 points)

2. Now, suppose that the production of public good relies on individuals' voluntary contributions. Each individual \( i \) is asked to contribute \( g^i \) of his initial endowment \( x^{*i} \) to public good production. Write the optimality condition for each individual. Show the existence of a "free riding" problem. (1 point)
3. Consider a great number of persons in society so that one can suppose
\[
\frac{\partial G}{\partial y^j} = 0
\]
Show that in this case, each individual may be a pure "free rider", i.e. choose a null contribution hoping to benefit from the expenditures of others. What is the equilibrium quantity of public good? (1 point)

Exercise 6 (4 points)
Consider an agency relationship in which the principal contracts the agent, whose effort determines the result. Assume that the agent can exert two effort levels \( e \), high or low, which induce a production error with probability 0.25 and 0.75 respectively. His utility function is
\[
U(w, e) = u(w) - v(e) = 100 - 10/w - v
\]
where \( w \) is the wage received and \( v \) takes the value 2 if effort is high and 0 if effort is low. Production errors are observable but effort cannot. The product obtained is worth \( x_2 = 20 \) if there are no errors and \( x_1 = 0 \) otherwise. Assume that the agent has reservation utility equal to \( U = 0 \).
The principal is risk-neutral. His objective function is
\[
B(x, w) = x - w
\]
1. Assume that the effort is not observable and the principal desires that the agent exerts the high effort. Write the principal’s profit maximization program and the first order conditions. (1 point)

2. Show that participation constraint and incentive constraint bind at the optimal contract. Interpret the result. (2 points)
3. Calculate the optimal wage $w_1$ corresponding to $x_1 = 0$ and the optimal wage $w_2$ corresponding to $x_2 = 20$. (1 point)