Microeconomic Theory
September 2011 VCREME Pre-Masters Program
Final Exam (Closed Book):

- The exam has two parts, A and B. For each part, Answer any 2 out of the 3 questions.
- Each question in part A is worth 10 points.
- Each question in part B is worth 20 points.
- Explain your answers and illustrate them if this helps.
- Time allowed for exam: 3 Hours (I expect you to finish the exam in under 2.5 hours) hours.

**Section A: Do any 2 of the following 3 questions.** Explain your answers to the fullest extent possible.

1. What is the *Envelope Theorem*? Give one of its applications in microeconomic theory.

2. What is a *Fixed Point Theorem*? Explain why it is central in understanding equilibrium in microeconomic theory.

3. What is the *First Theorem of Welfare Economics*? What is its significance to economics.

**Section B: Do any 2 of the following 3 questions.**

1. Consider a consumer who has the following indirect utility function: $$\frac{m}{p_1 + p_2}$$, where $m$ is the income, and $p_i$ is the price of good $i = 1, 2$.
   (a) Find the (Marshallian) demand functions for the two goods.
   (b) What is meant by the *expenditure function*? Find the expenditure function for this consumer.
   (c) What is meant by the *Hicksian* (or compensated) function? Find the Hicksian demand for the two commodities.
(d) What is the Slutsky equation? Show that it holds for this consumer.

(e) From the information you have obtained, what can you say about the utility function of this consumer? In particular, is the consumer willing to substitute between commodities?

2. Consider a pure-exchange economy with two consumers, \( i = 1, 2 \). There are two goods \( x, y \) and consumer \( i \)'s consumption of the goods is denoted by \( x_i, y_i \). The endowments of the two consumers, \( \omega_i \) is given by \( \omega_1 = (2, 1) \) and \( \omega_2 = (1, 2) \). The preferences for the two consumers are: 
\[
    u_1 = \alpha \ln x_1 + (1 - \alpha) \ln y_1, \quad 0 < \alpha < 1, \quad \text{and} \quad u_2 = \beta \ln x_2 + (1 - \beta) \ln y_2, \quad 0 < \beta < 1.
\]

(a) Define a Competitive (or Walrasian) Equilibrium.

(b) Find the demand functions for the two commodities for the two consumers.

(c) What is the Competitive Equilibrium price for this economy? Why can we solve only for the relative price of the two commodities?

(d) Define the Core of an economy.

(e) Why will the Competitive Equilibrium for this (or any economy) be in the Core? (It is sufficient to argue this for a 2-person economy such as this one).

3. Consider the game of chicken. There are two players, \( V \) and \( C \) who are driving on a one lane highway. \( C \) drives north to south, and \( V \) drives south to north. As they approach each other, if one swerves off the road to avoid the collision then it loses face and the other wins; if both swerve to avoid the collision then both lose face; if neither swerves then there is a collision and both are killed. Thus, each the strategy of Swerve (\( S \)) and Dont Swerve (\( DS \)). The payoff matrix is given as follows, where \( V \) is the row player and \( C \) is the column player. The entry \( (a,b) \) in a cell of the matrix are the payoffs to \( V \) and \( C \) respectively.

<table>
<thead>
<tr>
<th></th>
<th>DS</th>
<th>S</th>
</tr>
</thead>
<tbody>
<tr>
<td>DS</td>
<td>-3,-3</td>
<td>2,0</td>
</tr>
<tr>
<td>S</td>
<td>0,2</td>
<td>1,1</td>
</tr>
</tbody>
</table>
(a) Define a Nash equilibrium. What is a pure strategy Nash equilibrium? What is a mixed strategy Nash equilibrium.
(b) Explain why while a pure strategy Nash equilibrium for a game may not exist, a mixed strategy Nash equilibrium will.
(c) Find all the pure strategy Nash equilibria for this game.
(d) Find all the mixed strategy Nash equilibria for this game.
(e) What is the probability that both the players will survive at the end of the game.